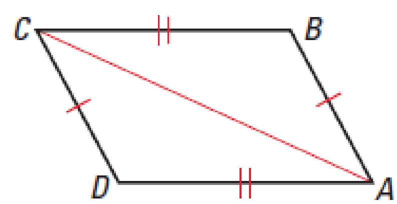
Proof Packet Answer Key

Proof of theorem 6.6

Prove Theorem 6.6.

GIVEN ▶ $\overline{AB} \cong \overline{CD}, \overline{AD} \cong \overline{CB}$

PROVE ▶ $ABCD$ is a parallelogram.



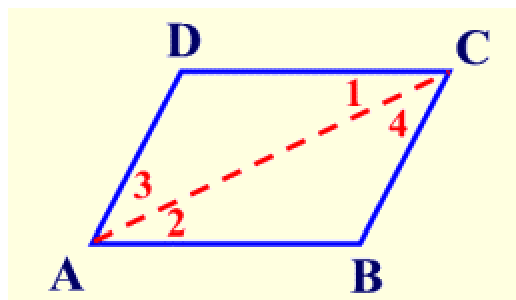
Statements	Reasons
1. $\overline{AB} \cong \overline{CD}, \overline{AD} \cong \overline{CB}$	**1.** Given
2. $\overline{AC} \cong \overline{AC}$	**2.** Reflexive Property of Congruence
3. $\triangle ABC \cong \triangle CDA$	**3.** SSS Congruence Postulate
4. $\angle BAC \cong \angle DCA$, $\angle DAC \cong \angle BCA$	**4.** Corresponding parts of $\cong$ ▵ are $\cong$.
5. $\overline{AB} \parallel \overline{CD}, \overline{AD} \parallel \overline{CB}$	**5.** Alternate Interior Angles Converse
6. $ABCD$ is a ▱.	**6.** Definition of parallelogram

Two pairs of opposite sides parallel proof

Given: parallelogram ABCD

Prove: $\overline{AB} \cong \overline{CD}$

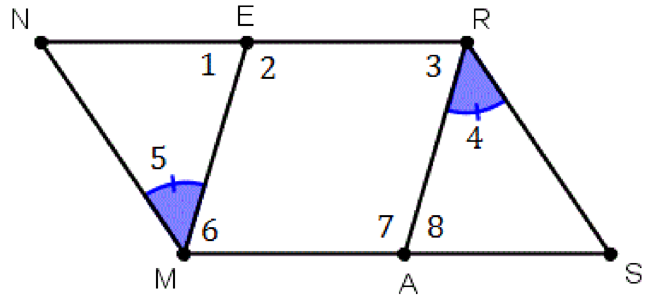
$\overline{BC} \cong \overline{AD}$

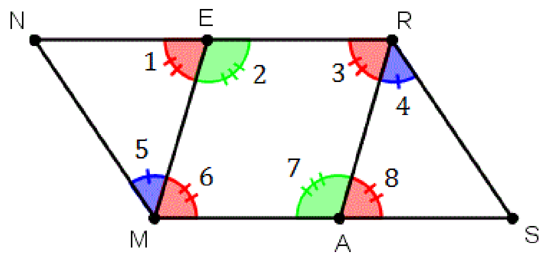


	STATEMENTS		REASONS
1	$\overline{AB} \cong \overline{CD}; \overline{AB} \parallel \overline{CD}$	**1**	Given
2	Draw segment from A to C	**2**	Two points determine exactly one line.
3	$\measuredangle 1 \cong \measuredangle 2$	**3**	If two parallel lines are cut by a transversal, the alternate interior angles are congruent.
4	$\overline{AC} \cong \overline{AC}$	**4**	Reflexive property: A quantity is congruent to itself.
5	$\Delta ABC \cong \Delta CDA$	**5**	SAS: If two sides and the included angle of one triangle are congruent to the corresponding parts of another triangle, the triangles are congruent.
6	$\measuredangle 3 \cong \measuredangle 4$	**6**	CPCTC: Corresponding parts of congruent triangles are congruent.
7	$\overline{AD} \parallel \overline{BC}$	**7**	If two lines are cut by a transversal and the alternate interior angles are congruent, the lines are parallel.
8	$\square ABCD$	**8**	A **parallelogram** is a quadrilateral with both pairs of opposite sides parallel.

Parallelogram inside a parallelogram proof

Given : $NRSM$ is a parallelogram
$\angle 4 \cong \angle 5$
Prove : $ERAM$ is a parallelogram





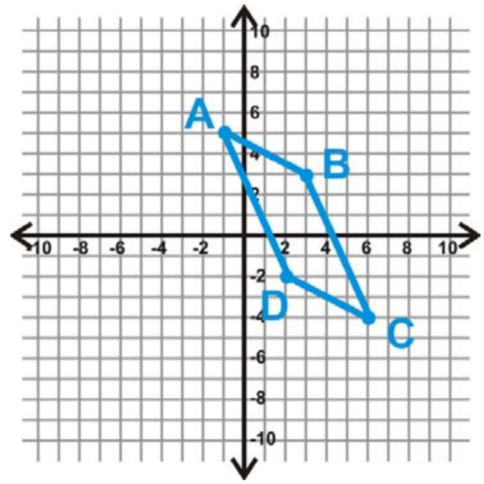
Statements	Reasons
1. $\angle 4 \cong \angle 5$	1. Given
2. $\angle R \cong \angle M$	2. The opposite angles of a parallelogram are congruent.
3. $\angle 3 \cong \angle 6$	3. Angle Subtraction Postulate
4. $NRSM$ is a parallelogram	4. Given
5. $\overline{NR} \parallel \overline{MS}$	5. The opposite sides of a parallelogram are parallel.
6. $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 8$	6. Alternate Interior Angles Theorem
7. $\angle 1 \cong \angle 8$	7. Transitive Property
8. $\angle 2 \cong \angle 7$	8. Congruent Supplements Theorem
9. $ERAM$ is a parallelogram	9. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Coordinate plane proof

Given: coordinate drawing to the right.

Prove: ABCD is a parallelogram.

Hint: use a theorem that involves congruent sides and the distance formula and/or parallel sides and the slope formula.



Solution: We have determined there are four different ways to show a quadrilateral is a parallelogram in the $x - y$ plane. Let's use Theorem 6-12 First, find the length of AB and CD.

$$AB = \sqrt{(-1-3)^2 + (5-3)^2} \qquad CD = \sqrt{(2-6)^2 + (-2+4)^2}$$
$$= \sqrt{(-4)^2 + 2^2} \qquad = \sqrt{(-4)^2 + 2^2}$$
$$= \sqrt{16+4} \qquad = \sqrt{16+4}$$
$$= \sqrt{20} \qquad = \sqrt{20}$$

$AB = CD$, so if the two lines have the same slope, $ABCD$ is a parallelogram.

Slope $AB = \dfrac{5-3}{-1-3} = \dfrac{2}{-4} = -\dfrac{1}{2}$ Slope $CD = \dfrac{-2+4}{2-6} = \dfrac{2}{-4} = -\dfrac{1}{2}$

By Theorem 6-12 $ABCD$ is a parallelogram.